How Does Inflation Affect Different Age Groups?*

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Abstract

This paper proposes an overlapping-generations model with sticky wages and prices to study which inflation rate is socially optimal. We show that, in combination with sticky wages, changes in productivity over workers' lives have important consequences for the socially optimal inflation rate. If we take this feature into account, we obtain that moderate deflation is optimal in contrast with the positive levels of inflation that would otherwise be optimal. We also study intergenerational conflicts and show that young voters support a transition to lower inflation while older ones tend to support higher inflation rates.

Keywords: Trend inflation, inflation target, sticky wages, sticky prices, overlapping-generations models.

JEL: D25, E31, E52, D72.

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1 Introduction

This paper analyzes how trend inflation affects different age groups. Moreover, we revisit the question of which trend inflation rates are optimal and ask how the answer to this question differs across age groups. For this purpose we study an overlapping-generations model with wage and price stickiness as well as three sources of productivity changes: (i) aggregate productivity growth, (ii) productivity growth for individual firms, and (iii) productivity changes of individual workers as they grow older. Under flexible prices and wages, these changes in productivities would lead to rather complex changes in relative prices over time. Sticky wages and goods prices distort relative prices compared to the benchmark with flexible price and wages. The magnitude of these distortions is affected by trend inflation in non-trivial ways.

When we abstract from wage rigidity, our model implies positive optimal rates of inflation of around 2%, in line with a finding in the literature that, under sticky prices, positive trend inflation can be desirable as it automatically implies that older and more productive firms charge lower relative prices than young and less productive firms (Adam and Weber, 2019a). One of our main quantitative findings is that, when wages are sticky as well, age-dependent productivity together with aggregate growth can overturn this effect and can make deflation with an inflation rate of -3.75% optimal. Intuitively, for young and middle-aged workers, individual labor productivity increases over time. This entails that, under flexible prices and wages, real wages would increase as workers grow older. Negative rates of inflation allow to implement real wage decreases even in the absence of nominal wage adjustments. Thus our results are qualitatively in line with Amano et al. (2009), who also find deflation to be optimal in the presence of sticky wages and aggregate productivity growth.

As younger workers have the steepest productivity increases over time, they tend to benefit more from low inflation rates than other workers. To investigate intergenerational conflicts in more detail, we extend our model by incorporating a political dimension. Starting from an initial steady state of the economy with a given level of trend inflation, all individuals can vote in favor of or against a moderate permanent change in inflation, which would cause a transition to a new steady state. We find that 2% deflation would correspond to a politico-economic equilibrium, as a majority of voters would prefer not to raise or lower inflation in this case.[1] Hence the political process tends to result in an inflation rate that is higher than the socially optimal one. Thus our paper identifies a new source of inflation bias which is complementary to the traditional one that is due to time-inconsistent policies (Kydland and Prescott, 1977; Barro and Gordon, 1983).

Our model involves quite complex mechanisms regarding the consequences of inflation for aggregate variables and wealth accumulation over the life cycle. These model outcomes are driven strongly by wage choices. A key observation is that positive inflation rates induce workers to choose high wage markups in order to avoid low real wages in the future, should they be unable to adjust their wage for some time (see Amano et al., 2009). This effect is particularly pronounced for young workers, who would prefer real wages that rise over time, in line with their anticipated high gains in individual productivity. This has several implications.

First, high wage markups under high inflation tend to lead to a low level of employment. Second, aggregate productivity is high for high inflation rates due to a composition effect. Older workers with high levels of labor productivity work more compared to the relatively unproductive young. Third, high wage markups for young workers result in low labor incomes, which leads to less accumulation of wealth up to retirement and a lower stock of capital. Fourth, high inflation rates involve large adjustments of nominal wages by young workers whenever they are able to adjust their wages. Thus high inflation rates are associated with a high degree of income uncertainty. Fifth, there are two channels via which the marginal product of capital is affected. Aggregate productivity is high for high inflation rates. Moreover, high inflation leads to a drop in employment, which increases the marginal product of capital. As a consequence, higher inflation rates are associated with higher real interest rates. The higher interest rates are attractive for middle-aged and old individuals for whom capital income is important.

When we analyze which inflation rate is chosen in the political process, we also take transition dynamics into account, which involves another channel that works through the stock market. A transition to higher inflation leads to lower output and profits and thereby to a sizable drop in stock prices. The drop in stock prices induces retirees and workers close to retirement, who are comparably wealthy, to oppose inflation rates higher than -2% despite the resulting increases in real interest rates that are desirable from their perspectives.

[1] A related politico-economic equilibrium in an overlapping-generations model is studied in Gonzalez-Eiras and Niepelt (2008), who focus on social security.

Our work contributes to the large literature on optimal inflation in the long run. (see Schmitt-Grohé and Uribe, 2010; Ascari and Sbordone, 2014; Diercks, 2017, for surveys). A classic argument stresses that higher inflation is associated with higher nominal interest rates and thus larger opportunity costs of holding money (see e.g. Fischer, 1981; Lucas, 1981). As a consequence, the Friedman rule, i.e. permanent deflation that eliminates the opportunity costs of holding money, is optimal. By contrast, zero inflation is typically welfare-maximizing in the standard new Keynesian model with sales subsidies in the limiting case where real money balances are zero, as it alleviates the distortions in relative prices under staggered price setting.[2] The consequences of a lower bound on nominal interest rates for the optimal rate of inflation are analyzed by Coibion et al. (2012) and Blanco (2021). Our analysis does not take into account the zero lower bound explicitly. However, nominal interest rates are always above 3% and thus the effective lower bound would never bind. Nevertheless it is plausible that an effective lower bound together with the possibility of large aggregate shocks would call for higher inflation targets in our framework. At any rate, due to the ongoing trend towards cashless societies and the introduction of central-bank digital currency, the effective lower bound may be less restrictive for monetary policy in the near future.

Second, our analysis is related to papers that study changes in trend inflation in models with productivity growth (Amano et al., 2009; Adam and Weber, 2019a, 2019b; Adam et al., 2022). Adam and Weber (2019a) propose a model where goods prices are sticky and individual firms become more productive over time. In this case, inflation of around 2% is optimal because it allows for relative prices to reflect the relative productivities of different generations of firms even if nominal prices are never adjusted. We extend Adam and Weber's analysis by taking sticky wages as in Amano et al. (2009) and, in addition, productivity changes over workers' life cycles into account.

Third, while new Keynesian business cycle analysis typically relies on models that focus on sticky prices of goods, some authors argue that sticky wages may be even more important for understanding the dynamic effects of shocks (Erceg et al., 2000; Christiano et al., 2005; Amano et al., 2009; Broer et al., 2020). By contrast, Basu and House (2016) highlight that the wage measure that is relevant for employment dynamics (e.g. the user cost of labor proposed by Kudlyak, 2014)

[2]The costs of price dispersion may be exaggerated by models with Calvo pricing that assume that firms cannot ration demand even if this was profitable to them (Hahn, 2022).

is remarkably flexible. Gertler et al. (2020) criticize this view and argue that the measures of the user cost of labor employed by Kudlyak (2014) and Basu and House (2016) may be biased and that the true user cost of labor may be less cyclical than found by them. With the help of a micro data set on wages in Sweden, Björklund et al. (2019) show that nominal wage rigidity is important for understanding the real effects of monetary policy. Thus it may be plausible that sticky wages are also relevant for the welfare effects of different levels of trend inflation.

Our paper is organized as follows. The subsequent section lays out our model and specifies the parameter values for the numerical analysis. Section 3 highlights the effects of different trend inflation rates for markups, aggregate variables, and individual consumption. Moreover, we flesh out the intergenerational conflicts and identify the consequences of the level of steady-state inflation for welfare. Section 4 analyzes transition dynamics and provides our results about politico-economic equilibria, where individuals vote on permanent changes in trend inflation. We present our conclusions in section 5.

2 Model

2.1 Set-up

We consider an overlapping-generations model with sticky prices and wages. The economy is populated by workers, retirees, intermediate-goods producers, perfectly competitive final-goods producers, and a monetary authority.

There are T generations of workers, where age is denoted by $\tau = 1, 2, ..., T$. After reaching age T, individuals are retired for TR periods. They die after reaching age $\tau = T + TR$ and are replaced by new workers of age $\tau = 1$. The size of the population is normalized to one. We use $\lambda = \frac{T}{T+TR}$ to denote the share of workers in the population.

Worker i's utility in period t is

$$u(C_{i,t}, H_{i,t}) = \ln(C_{i,t}) - \eta \frac{H_{i,t}^{1+\kappa}}{1+\kappa}, \tag{1}$$

where η is a positive parameter and κ the inverse Frisch elasticity of the labor supply. $H_{i,t}$ is the number of hours worked, $C_{i,t}$ final-goods consumption. Utility in future periods is discounted by factor $\beta \in (0, 1)$.

Workers i's individual productivity is $G_{i,t}$, which is a function of age $\tau_{i,t}$:

$$G_{i,t} = g(\tau_{i,t}) \tag{2}$$

Effective labor $L_{i,t}$ and the number of hours worked by worker i, $H_{i,t}$, are related by $L_{i,t} = G_{i,t}H_{i,t}$. $W_{i,t}$ is the hourly nominal wage. In every period, workers are unable to adjust their nominal wage with probability $\omega \in [0, 1]$. Young workers of age 1 can choose their wages freely.

There are two different assets that workers can hold: physical capital $K_{i,t}$ with a rental rate r_t and stocks $s_{i,t}$, which have an ex-dividend price Q_t. The stocks are claims on the profits Π_t of intermediate-goods producers. The stock price is

$$Q_t = \sum_{s=1}^{\infty} \prod_{j=1}^{s} \frac{1}{1 + r_{t+j} - \delta} \Pi_{t+s}. \tag{3}$$

The aggregate supply of stocks is one.

In the steady state, both assets have identical returns as they are perfect substitutes. Later we will also examine unexpected shocks to the economy. In this case, the returns may be different ex post. Both assets are held in identical proportions by all individuals. This assumption does not affect our steady-state results but will be relevant for the effects of unanticipated shocks. Young workers of age $\tau = 1$ have zero assets when entering the economy.

Worker i's budget constraint is

$$C_{i,t} + K_{i,t+1} + s_{i,t+1}Q_t = (1 + r_t - \delta)K_{i,t} + \frac{W_{i,t}}{P_t}H_{i,t} + s_{i,t}\Pi_t + s_{i,t}Q_t. \tag{4}$$

Retirees have the same utility functions and budget constraints as workers but cannot work, i.e. $H_{i,t} = 0$. They use their asset holdings to finance consumption. All individuals are borrowing constrained. More specifically, each individual i's total asset holdings can never be negative.

Perfectly competitive final-goods producers combine intermediate goods produced by firms $f \in [0, 1]$ to a final good according to the technology

$$Y_t = \left[\int_0^1 Y_{f,t}^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{5}$$

where $\varepsilon > 1$ is the elasticity of substitution. Final goods can be used both for consumption and for investment. As is well-known, the final-goods producers' optimization problem entails that the demand for firm f's intermediate good is

$$Y_{f,t} = \left(\frac{P_{f,t}}{P_t}\right)^{-\varepsilon} Y_t, \tag{6}$$

where $P_{f,t}$ is the nominal price chosen by firm f and the price level P_t satisfies

$$P_t = \left[\int_0^1 P_{f,t}^{1-\varepsilon} df\right]^{\frac{1}{1-\varepsilon}}. \tag{7}$$

Each intermediate-goods producer f rents capital and hires the different varieties of labor to produce intermediate-good variety f. The production function is

$$Y_{f,t} = A_t X_{f,t} K_{f,t}^{\alpha} L_{f,t}^{1-\alpha}, \tag{8}$$

where A_t is aggregate productivity and $X_{f,t}$ firm-specific productivity. With slight abuse of notation, $K_{f,t}$ denotes the amount of capital rented by the firm and $L_{f,t}$ is the composite labor employed by firm f (more details on this later). In period t, intermediate-goods producers use $D_{t,t+s} := \sum_{s=1}^{\infty} \prod_{j=1}^{s} \frac{1}{1+r_{t+j}-\delta}$ to discount profits in period $t+s$ $(s = 1, 2, ...)$.

Aggregate productivity A_t evolves according to $A_t = aA_{t-1}$ with $a \geq 1$ and $A_0 = 1$. Following Adam and Weber (2019a), we assume that firm-specific productivity $X_{f,t}$ increases with firm age, i.e. $X_{f,t+1} = qX_{f,t}$ with $q > 1$. With constant probability $d \in (0, 1]$, firm f exits the market at the end of each period and is replaced by a new firm. A new firm starts with $X_{f,t} = 1$. In each period, a firm has to keep the previous period's nominal price with probability $\phi \in [0, 1]$. It is able to adjust it with probability $1 - \phi$. New firms can choose their prices freely.

Composite labor employed by firm f is given by

$$L_{f,t} = \left[\int_0^{\lambda} L_{f,i,t}^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}},$$

where $\theta > 0$ and $L_{f,i,t}$ is the amount of labor from worker i that is hired by firm f.

Firm f's cost-minimization problem entails that firm f's demand for effective labor of type i is

$$L_{f,i,t} = \left[\frac{W_{i,t}}{G_{i,t} W_t}\right]^{-\theta} L_{f,t}, \tag{9}$$

$$W_t = \left[\int_0^\lambda \left(\frac{W_{i,t}}{G_{i,t}} \right)^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \tag{10}$$

As a consequence, the demand for worker i's raw labor is

$$H_{i,t} = (G_{i,t})^{\theta-1} \left(\frac{W_{i,t}}{W_t} \right)^{-\theta} L_t, \tag{11}$$

where $L_t := \int_0^1 L_{f,t}\, df$. In addition, asset markets have to clear in every period t:

$$K_t = \int_0^1 K_{i,t-1}\, di \;=\; \int_0^1 K_{f,t}\, df \tag{12}$$

$$\int_0^1 s_{i,t}\, di \;=\; 1. \tag{13}$$

Final-goods market clearing implies

$$C_t + K_{t+1} = Y_t + (1-\delta)K_t. \tag{14}$$

The monetary authority conducts monetary policy in a way such that inflation is fixed at π. For example, monetary policy could be implemented via an interest-rate rule.

The equilibrium concept is standard. In every period, workers choose consumption, asset holdings for the next period, and, in periods where they can adjust wages, nominal wages subject to their budget constraints and borrowing constraints to maximize the present value of current and future per-period utility. Their individual state variables are age, wealth, and the nominal wage inherited from the previous period (unless they are able to adjust the wage). The optimization problem of the retirees is analogous with the only difference that they cannot work and thus there is also no nominal wage for them. Final-goods producers select optimal bundles of intermediate goods, taking the prices of these intermediate goods as given. This leads to the demand for intermediate goods specified in (6). Intermediate-goods producers choose optimal amounts of capital and varieties of labor and, whenever possible, optimal prices of their goods in order to maximize the present value of their profits. The individual state variables are firm-specific productivity $X_{f,t}$ and, in periods where price adjustment is not possible, the previous period's price. The optimal choices of labor entail (11). Moreover, asset markets and goods markets have to clear. More details on the firms' and workers' optimization problems, which lead to the optimal price and wage choices, can be found in Appendix A.

In order to analyze steady states, it is useful to recognize that real variables like aggregate output, consumption, and labor grow at rate $a^{\frac{1}{1-\alpha}}$ in the long run. Thus we introduce detrended variables by dividing by $\left(a^{\frac{1}{1-\alpha}}\right)^t$. Detrended variables are denoted by $\sim$. In a steady-state, detrended real variables are constant over time.

2.2 Aggregate output

In the following, we discuss how aggregate output is determined in equilibrium, which will be useful for the discussion of our results later. Combining (6) and (8), taking into account that $K_{f,t}/L_{f,t} = K_t/L_t$ holds for all firms f, and integrating yields

$$Y_t = A_t A_t^G K_t^\alpha L_t^{1-\alpha}, \tag{15}$$

where A_t^G is the inverse of the measure of price dispersion often used in new Keynesian models (see e.g. Ascari and Sbordone, 2014):

$$A_t^G := \left(\int_0^1 \frac{1}{X_{f,t}} \left(\frac{P_{f,t}}{P_t} \right)^{-\varepsilon} df \right)^{-1}. \tag{16}$$

Because A_t^G measures how efficiently resources are allocated across intermediate-goods producers, we label A_t^G goods-market efficiency.

Next we consider the relationship between aggregate hours worked $H_t := \int_0^\lambda H_{i,t}\, di$ and composite labor L_t. With the help of (11), we obtain

$$L_t = \left(A_t^L\right)^{\frac{1}{1-\alpha}} H_t, \tag{17}$$

where we have introduced labor-market efficiency A_t^L as

$$A_t^L := \left(\int_0^\lambda \left(G_{i,t}\right)^{\theta-1} \left(\frac{W_{i,t}}{W_t} \right)^{-\theta} di \right)^{-(1-\alpha)}. \tag{18}$$

Labor-market efficiency is, up to a monotonic transformation, the inverse of the standard measure of wage dispersion used in sticky-wage models. Labor-market efficiency describes the efficiency of the allocation of the different types of labor due to staggered wage-setting.

The aggregate production function can be written as

$$Y_t = A_t A_t^G A_t^L K_t^\alpha H_t^{1-\alpha}. \tag{19}$$

Thus output depends not only on exogenous aggregate productivity A_t, capital K_t, and employment H_t but also on endogenous goods-market efficiency A_t^G and labor-market efficiency A_t^L. We will call the product $A_t^G A_t^L$ aggregate efficiency in the following.

In a steady state, the aggregate production function can be conveniently expressed with the help of detrended ("$\sim$") variables:

$$\tilde{Y} = A^G A^L \tilde{K}^\alpha H^{1-\alpha}. \tag{20}$$

2.3 Parameter values

The model is parameterized on a quarterly basis. Table 1 gives an overview over all parameter values. We follow Amano et al. (2009) by choosing an annual growth rate of aggregate productivity of 2%. The depreciation rate of capital δ is 2.5% (Ascari et al., 2018). We set the price elasticity of substitution, ε to 7 (in line with Adam and Weber, 2019; Nakamura and Steinsson, 2008). The firm exit probability is estimated by Adam and Weber (2019a) to be around 2.9%. This value corresponds to the average firm birth and exit rate in U.S. firm data from the Business Dynamics Statistics (BDS). To determine q, the productivity growth rate for individual firms, we use BDS data and compute the employment growth for existing firms over the years 2000-2018. Then we calculate the average and weight observations by the numbers of firms' establishments. The resulting growth rate of 2.4% p.a. pins down the value of q.

For the elasticity of substitution between different types of labor, we set θ to 6, which is in the range of values used in the literature, reaching from 4 to 21 (Christiano et al., 2005; Amano et al., 2009; Erceg et al., 2000). Our value corresponds to the value chosen by Ascari et al. (2018). We assume a working-life horizon of 45 years (180 periods) and that individuals start their working life at the age of 20. After retirement, at the age of 65, individuals live for another 14 years and die thereafter, which means that the life expectancy corresponds to the one in the US in 2019, i.e. before the Covid-19 pandemic.[3] For the age-dependent labor

[3]Source: `https://genderdata.worldbank.org/`.

Parameter	Value	Source
Aggregates		
a	$1.02^{(1-\alpha)0.25}$	Amano et al. (2009)
δ	0.025	Ascari et al. (2018)
Firm		
ε	7	Firm Mark-up=$\frac{1}{\varepsilon-1}$, Adam and Weber (2019a), Nakamura and Steinsson (2008)
d	0.029	Average of firm birth and exit rate (Adam and Weber, 2019a)
α	$\frac{1}{3}$	Capital-output share
q	$exp(1.024^{0.25}-1)$	Firm productivity growth trend, Source: BDS data
Individuals		
θ	6	Ascari et al. (2018)
T	180	45 years working life
TR	56	14 years retirement
$g(\tau_{i,t})$	Hanse (1993)	Worker's productivity growth
β	0.99	Standard value
η	1	Dotsey et al. (1999); Golosov and Lucas (2007)
κ	1	Inverse of Frisch elasticity (Amano et al., 2009)
Nominal frictions		
ϕ	0.55	Adam and Weber (2019a), Coibion et al. (2012)
ω	0.75	Amano et al. (2009)

Table 1: Parameter values

productivity profile we use standard estimates by Hanse (1993). We normalize the productivity profile such that 20-year-old workers start their working lives with an individual productivity level of 1. Figure 1 shows the annualized growth rate of individual productivity. It is very high for young workers, declines as workers become older and turns negative for workers who are nearly fifty years old.

We set the quarterly discount factor to 0.99. Concerning the parameters governing the disutility of labor in workers' utility, we set both, η and κ to 1 (compare Dotsey et al., 1999; Golosov and Lucas, 2007; Amano et al., 2009). A value of $\phi = 0.55$ implies that nominal prices are adjusted every 7 months on average (compatible with Adam and Weber, 2019; Coibion et al., 2012). To match the empirically observed frequency of annual wage adjustments, we set ω to 0.75 (see Amano et al., 2009).

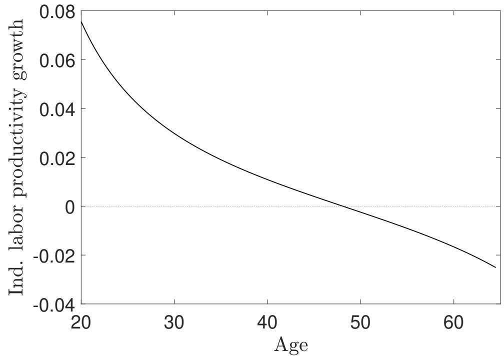

Figure 1: Annualized growth rate of individual labor productivity as a function of age.

The algorithm that is used to compute the steady states is described in Appendix B.

3 Effects of Different Trend Inflation Rates

3.1 Overview

In the following, we present simulation results for steady states with different inflation rates and examine the consequences of different trend inflation rates for individual prices and wages as well as aggregate variables. We start with an analysis of the price markups of goods (Section 3.2) and proceed with a discussion of wage markups (Section 3.3). One key finding is that higher trend inflation tends to lead to substantially higher markups in the labor market. This has important consequences for aggregate employment and other aggregate variables, which are considered in Section 3.4. In the subsequent Section 3.5, we discuss the implications of trend inflation for individual uncertainty about real wages, which arises due to the Calvo friction in the labor market. Finally, we will be in a position to discuss the implications of trend inflation for different age groups.

3.2 Markups in goods markets

Before we begin our analysis of markups in goods markets and their relation to trend inflation in our full model, it is useful to consider the case of flexible prices. Because firms' productivity grows at rate q as they become older, relative prices of individual firms deteriorate at a net rate of $q^4 - 1 \approx 2.4\%$ every year under flexible prices.

If the aggregate price level increases at this rate every year, the decline in relative prices can be obtained even when all firms never adjust their nominal prices. Thus, for a rate of inflation of 2.4%, sticky goods prices are not distortionary and the highest possible degree of goods-market efficiency A^G can be achieved even in the presence of sticky prices (see Adam and Weber, 2019a).

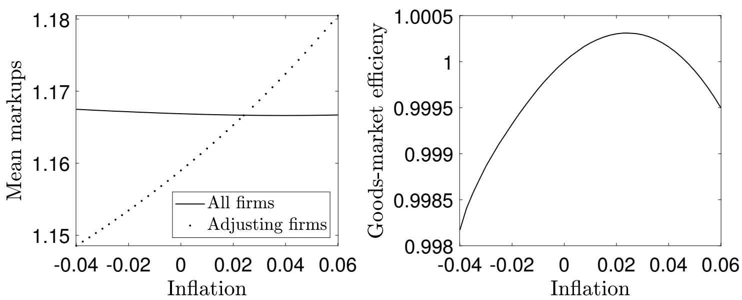

Figure 2: The effects of different trend inflation rates on the mean markups of all firms and adjusting firms (left panel) and on goods-market efficiency A^G (right panel) , where A^G has been normalized to one for $\pi = 0\%$.

These results are confirmed by Figure 2, which shows results from our simulations for different-steady state inflation rates under sticky goods prices. The left panel shows that, for a rate of inflation of 2.4%, the markups selected by firms that can adjust their prices are equal to $\varepsilon/(\varepsilon - 1) = 7/6 \approx 1.167$, which is the markup they would choose under flexible prices. At the same time, because firms never adjust their prices when inflation is 2.4%, the mean markups of all firms and the markups of adjusting firms are identical. The right-hand side panel confirms that goods-market efficiency is highest under the inflation rate under consideration.

For inflation rates higher than 2.4%, firms have to take into account that their markups deteriorate during periods where they are unable to adjust their prices. As a consequence, the markups of firms that adjust their prices are increasing functions of inflation. There are two opposing effects on mean markups. First, as we have discussed, newly selected markups are an increasing function of inflation. Second, markups for firms that cannot adjust their prices deteriorate over time when inflation is high. On balance, both effects almost cancel and mean markups decrease mildly with inflation (see the left panel of Figure 2).

3.3 Wage markups

If there were nominal rigidities only in goods markets but not in labor markets, the results discussed in the previous section would imply an optimal rate of inflation that is slightly higher 2.4% (in line with Adam and Weber, 2019a).[4] To examine the implications of sticky wages for our findings, it is instructive to consider a labor market with flexible wages first and to study how the real wage of an individual worker would evolve over time in this case. First, the real wage would increase as a result of aggregate growth. This effect in isolation would make a worker's wage grow at an annual rate of 2.0%. Second, one has to take into account that an individual worker's productivity changes over the life cycle. This effect would lead to high real wage growth for young workers and lower growth for older workers.

By solving our model for age-independent worker productivity, we are able to isolate the effects of aggregate growth for labor-market markups under sticky wages. The left panel of Figure 3 shows the wage markups of all workers and those of workers who are able to adjust their nominal wages as functions of trend inflation. At a negative inflation rate of -2%, workers' real wages grow at a rate of 2% during times where they cannot change their wages. As this growth rate of real wages implies constant markups, nominal wages are not changed even in periods where workers could change them. Thus newly adjusted wages equal mean wages. Moreover, markups always correspond to the markups under flexible wages ($\theta/(\theta - 1) = 6/5 = 1.2$). To sum up, a negative inflation rate of -2%

[4]The optimal inflation would be slightly higher than 2.4% because of the monopolistic distortions in the goods markets, which are alleviated by higher trend inflation. Compare the mean markups of all firms in Figure 2, which decrease with trend inflation.

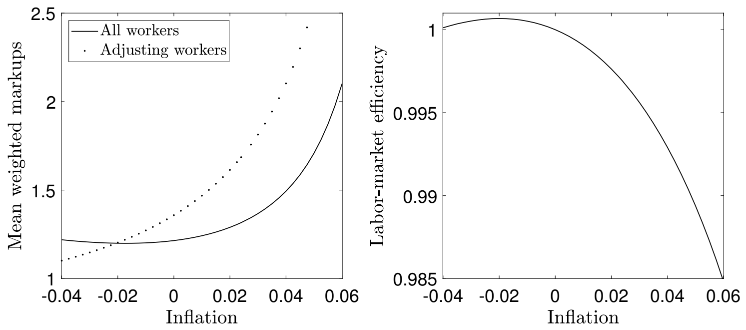

Figure 3: The effects of different trend inflation rates on the mean markups of all workers and adjusting workers (left panel) and on labor-market efficiency A^L (right panel) , where A^L has been normalized to one for $\pi = 0\%$. Special case where workers' productivity does not change over the life cycle.

guarantees that wage markups equal the wage markups that would prevail under flexible wages just like a positive inflation rate of 2.4% entails relative goods prices that emulate those under flexible prices. One would thus conjecture that, in the scenario where worker productivity does not depend on age, labor-market efficiency is highest under an inflation rate of -2. This is confirmed by the right panel of Figure 3.

As a next step, we turn to the full model, i.e. the more complex case where a worker's productivity depends on age. The corresponding results are displayed in Figure 4. The left panel focuses on young workers, who are aged 20-30 years and experience high labor-productivity growth. The panel shows the mean markups of all workers as well as all adjusting workers in this age group. The middle panel displays analogous graphs for middle-aged workers (aged 40-50), whose individual productivity growth stalls.

For the latter group of middle-aged workers, real wages would increase approximately at a rate of 2% under flexible wages, which is the growth rate of aggregate productivity. A negative inflation rate of -2% allows for real-wage growth of 2% even in the absence of nominal wage adjustments. As a consequence, mean wages and mean newly adjusted wages are approximately identical at this inflation rate, and wage markup dispersion is minimized within the group of middle-aged workers.

For young workers, real wages would grow at a substantially higher rate under flexible wages, namely at approximately 7%, which is the sum of the aggregate growth rate of 2% and a typical individual worker productivity growth rate of 5% in this age group (compare Figure 1). At negative inflation rates of approximately

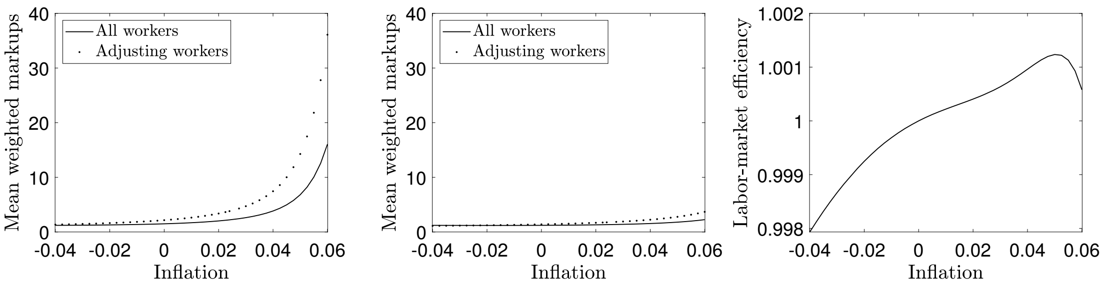

Figure 4: General case where workers' productivity changes over the life cycle. Left panel: the effects of different trend inflation rates on the mean markups of all young workers and adjusting young workers (aged 20-30). Middle panel: the effects of different trend inflation rates on the mean markups of all middle-aged workers and adjusting middle-aged workers (aged 40-50). Right panel: labor-market efficiency A^L as a function of trend inflation; A^L has been normalized to one for $\pi = 0\%$.

-7%, which is a bit smaller than the lowest rate of inflation displayed in the figure, nominal wages of young workers would thus be adjusted by only small amounts. This inflation rate would therefore minimize markup dispersion within this group.

The panel on the right-hand side shows that labor-market efficiency is maximized at an inflation rate of around 4.5%. This is due to the fact that labor-market efficiency does not only depend on markup dispersion within age groups but on the distribution of markups and productivities across age groups as well. As young workers' wage markups increase with inflation particularly strongly, high inflation rates imply that these comparably unproductive workers contribute relatively little to aggregate labor, which tends to make aggregate labor-market efficiency A^L high for moderate positive inflation.

It may be worth highlighting that the changes in labor-market efficiency in response to changes in trend inflation are larger than the corresponding changes in goods-market efficiency. Moreover, mean wage markups are more strongly influenced by inflation than the markups in goods markets. As discussed in Amano et al. (2009), the strong effect of inflation on wage markups is driven by an asymmetry in the utility function of workers. Wage markups that are too low compared to the markups under flexible wages lead to substantially larger utility losses than high markups.

The strong response of wage markups to inflation is a first indication that wage rigidities may be more important than price rigidities for understanding the consequences of trend inflation in our model. The marked rise in wage markups with inflation has sizable consequences for aggregate variables, as will be studied in more detail in the next section.

3.4 Aggregate variables

How key aggregate variables are affected by different trend inflation rates is shown in Figure 5. Aggregate efficiency, which is shown in the left panel of the first row,

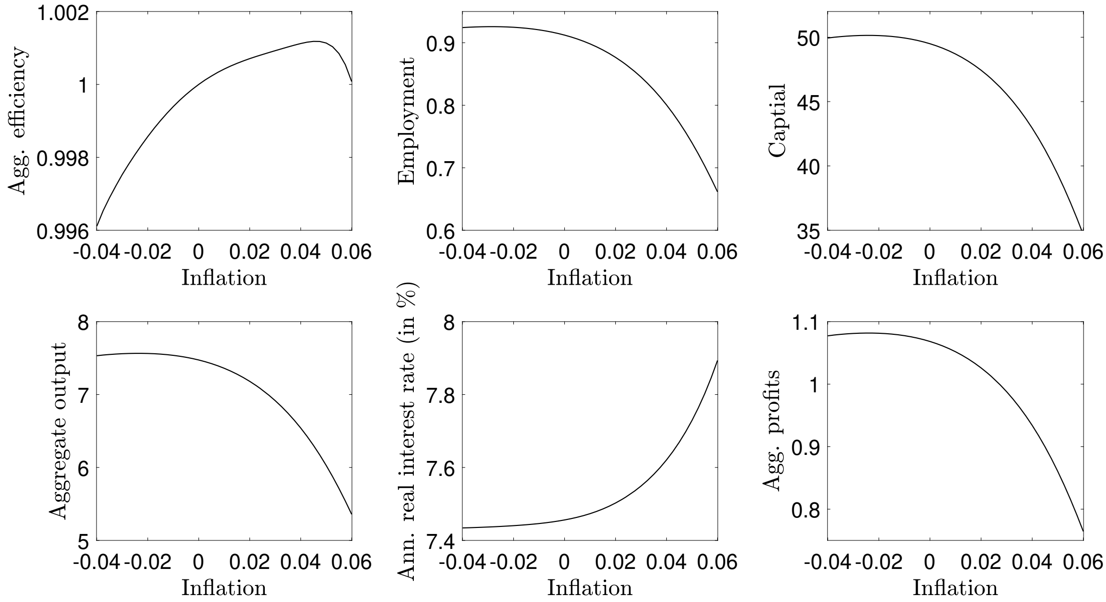

Figure 5: Aggregate variables for different trend inflation rates. First row: Aggregate efficiency (normalized to one at $\pi = 0\%$), aggregate hours worked, aggregate capital. Second row: output, annualized real interest rate, profits.

is the product of goods-market efficiency and labor-market efficiency. We have seen that goods-market efficiency has its maximum at 2% inflation (see Figure 2) and labor-market efficiency takes its highest value at inflation between 4% and 5%. As changes in labor-market efficiency are large around its maximum and goods-market efficiency is relatively flat around its maximum, the maximum of aggregate efficiency is located close to the maximum of labor-market efficiency at an inflation rate somewhat larger than 4%.

Section 3.3 has demonstrated that mean wage markups increase substantially with inflation. As a consequence, employment decreases quite strongly for higher rates of inflation (see the middle panel of the first row). The low levels of employment decrease the marginal product of capital and thus also the amount of capital rented by firms when inflation is high (see the right panel in the first row). The level of aggregate output as a function of inflation is dominated by the changes in employment and capital and hardly affected by the comparably modest changes in aggregate efficiency. As a result, aggregate output decreases substantially for higher inflation rates (see the left panel of the second row).

To lay the grounds for our analysis of the effects of changes in trend inflation for different age groups, it is instructive to examine changes in real interest rates and aggregate profits (see the middle and right panel in the second row of Figure 5). Real interest rates increase with inflation. This is due to the effect that high inflation rates imply high wage markups and thus high real wages. High real wages make it attractive for firms to substitute labor for capital to some extent, which tends to drive up real interest rates as well. As we will see, higher real interest rates are particularly attractive for retirees as they only receive capital income. Finally, we also observe that aggregate profits move more or less in lockstep with aggregate output. This is plausible because mean goods-markets markups are hardly affected by changes in inflation and thus profits are approximately proportional to aggregate sales or aggregate output (see Figure 2). Changes in profits will be important for understanding the impact of permanent changes in inflation on stock prices, which will be considered in section 4.

3.5 Individual uncertainty

In the course of our analysis, we have emphasized that nominal wage rigidity is far more important for understanding the consequences of trend inflation for aggregate economic variables than nominal price rigidities. There is a difference between sticky wages and sticky prices that we have not explored yet. Staggered wage adjustment causes uncertain individual labor income and thereby represents a source of uncertainty for individual workers whereas staggered price adjustment does not.

Figure 6 shows quartiles of the distributions of detrended real wages, wealth, and consumption for different age groups. We focus on two cases: deflation with an inflation rate of -4% and rather high inflation of 4%. Under deflation, real wages grow at a rate of roughly 4% when workers do not adjust their wages. As young workers' productivity increases approximately at this rate, wages are changed by only small amounts if workers have the opportunity to adjust them. As a result, the distribution of wages under deflation is relatively tight for young workers up to the age of 40. For older workers, productivity growth, which is the sum of aggregate growth a and the age-specific change in productivity, is smaller and below the 4% real wage growth caused by an inflation rate of 4%. This requires larger wage adjustments when old workers are able to change their wages and leads to a moderately larger wage dispersion within groups of identical age.

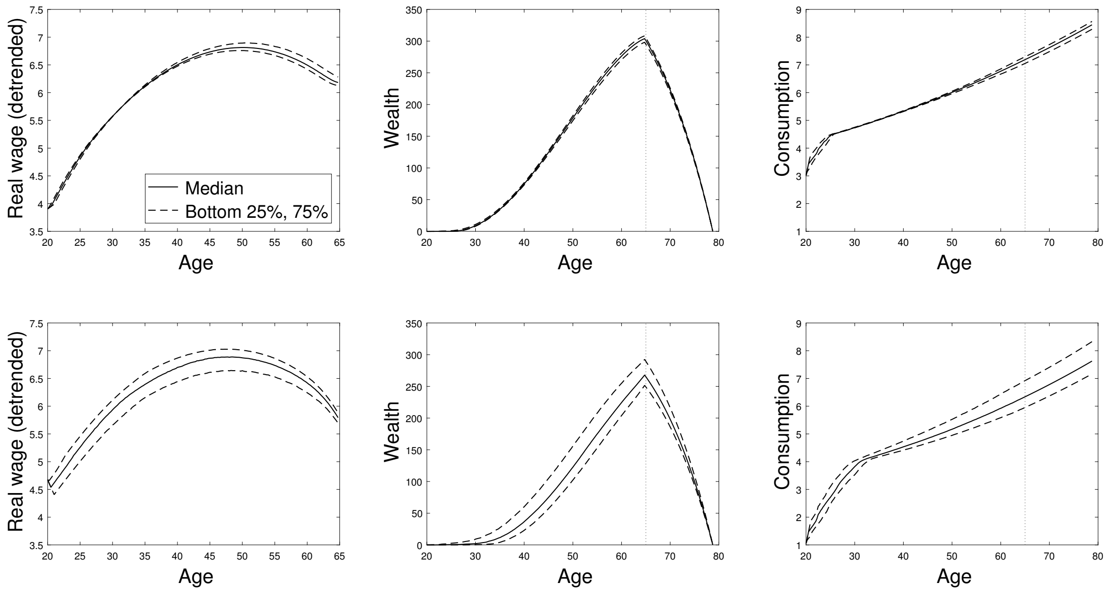

Figure 6: Quartiles of the distributions of individual real wages, wealth, and consumption for different age groups. First row: −4% inflation. Second row: +4% inflation.

Under an inflation rate of 4%, wages are substantially more dispersed across members of younger age groups compared to deflation. This follows from the substantial drops in real wages during periods of constant nominal wages, which induce younger workers to change their wages by large amounts whenever adjustments are possible. The dispersion of wages within groups of a specific age is comparably high even for old workers.

The larger dispersion of real wages under inflation has important consequences for the distributions of wealth and consumption among individuals of a specific age. The more dispersed real wages under high inflation lead to more dispersed incomes and thus more dispersed levels of wealth and consumption. Thus inflation causes much more individual uncertainty than deflation. Uncertain incomes are particularly harmful to young workers, who cannot borrow in order to dampen the consequences of low current incomes for current consumption.

Finally, we discuss how inflation affects the median levels of wages, wealth, and consumption of different age groups. Figure 6 shows that inflation influences the median level of consumption for young workers in particular. As has been demonstrated before, high inflation makes young individuals choose high wage markups as they have to factor in the possibility of declining real wages during periods of fixed nominal wages. This is also visible from the panels in the first column, which shows that young workers' real wages tend to be higher under inflation compared to deflation. High real wages tend to lead to low incomes and therefore low levels of consumption. For instance, in the presence of inflation, the consumption levels of 20-year-old workers are reduced to just one-third of what they are under deflation.

3.6 Preferences over trend inflation rates

Having discussed the consequences of inflation for various economic variables, we are now in a position to analyze which inflation rates would be preferred by different age groups. Before beginning with the analysis of transition dynamics, which is conducted in the subsequent section, we propose a comparably simple thought experiment. Consider an economy in a steady state with zero inflation. For each individual in this economy, we then ask which inflation rate from -4% to 6% would deliver the highest utility if the individual could, for given individual wage and wealth, move to an economy that is in the steady state with this different trend inflation rate but is identical with regard to all other exogenous parameters. For each age group, we then compute the median preferred inflation rate.

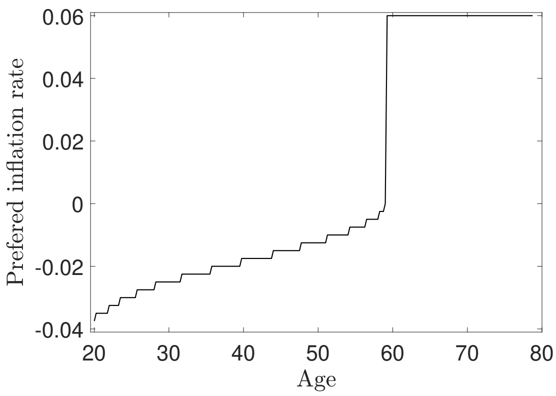

Figure 7: Median preferred inflation rates for different age groups (individual state variables drawn from distribution in the steady state with 0% inflation).

The median preferred inflation rates as a function of age are displayed in Figure 7. In line with our previous analysis, 20-year-old workers prefer deflation with negative inflation rates of almost -3.75%. Because workers' productivity growth is a decreasing function of age, older workers tend to prefer higher inflation rates. Individuals older than 60 years, who are retirees or at least close to retirement, are mainly interested in high real interest rates. As real interest rates increase with inflation, they prefer the highest inflation rate that we consider in this exercise.

As a next step, we study which inflation rate is socially optimal. For this purpose, we compare steady states and apply two different welfare measures. As a first measure, we employ the lifetime utility of the youngest individuals, who enter the economy. It may be important to note that all members of this group are identical, as opposed to older individuals, where wages and wealth differ across individuals of identical age. The socially optimal inflation rate in this case is identical to the inflation rate that is preferred by workers aged 20 in the previous thought experiment. Thus, according to our first welfare measure, deflation with an inflation rate of −3.75% is socially optimal.

Our second welfare measure corresponds to the mean of the instantaneous utilities of all individuals who are alive in a particular period in the steady state. This measure of welfare is similar in spirit to the unconditional expectation of a representative consumer's utility (or, equivalently, the unconditional loss), which is often used in new Keynesian models (see e.g. Rotemberg and Woodford, 1997, for an early contribution).

	lifetime utility at $\tau = 1$		avg. current utility	
Case	Opt. infl.	Welf. loss	Opt. infl.	Welf. loss
Main model	−3.75%	−3.65%	−3.00%	−2.30%
No wage stickiness $\omega = 0$	+2.75%	−0.06%	+2.75%	−0.07%
No price stickiness $\phi = 0$	−4.00%	−3.90%	−3.25%	−2.49%
No firm growth $q = 1$	−4.00%	−3.77%	−3.25%	−2.40%
No ind. labor growth $G = 1$	−1.75%	−0.61%	−1.75%	−0.64%
Agg. growth $a = 1.01^{(1-\alpha)0.25}$	−2.75%	−1.85%	−2.00%	−0.99%
No agg. growth $a = 1$	−2.00%	−0.74%	−0.75%	−0.09%
Frisch elasticity $\kappa^{-1} = 2$	−3.50%	−1.90%	−2.75%	−1.14%
Elasticity of sub. labor $\theta = 4$	−3.00%	−0.94%	−2.50%	−0.38%

Table 2: Optimal steady-state inflation rates and the welfare losses in terms of consumption equivalents implied by 0% inflation rather than the optimal inflation rates. Two welfare measures are considered: the expected lifetime utility of individuals who enter the economy at the youngest possible age and the mean instantaneous utility of all individuals currently alive.

Table 2 shows the socially optimal inflation rates for both welfare measures and different variants of our model, where we consider a grid of inflation rates with a step size of 0.25 percentage points. The welfare loss corresponds to the consumption equivalent if the economy is in the steady state of zero inflation rather than the optimal inflation rate. Our first observation is that our second welfare measure tends to lead to higher optimal trend inflation. This is plausible because, in line with our previous analysis, younger individuals typically prefer lower negative inflation rates than older individuals. The second welfare measure puts more weight on the utility of older individuals, which results in higher optimal rates of inflation.

A scenario of interest that is considered in Table 2 is the case without wage stickiness. In this case, we obtain results analogous to Adam and Weber (2019a), who find that positive inflation rates are optimal under sticky prices if individual firms become more productive over time and firms' relative prices should decrease accordingly. The socially optimal inflation rate in the case considered here is even a bit higher than the growth rate of firm productivity. This is due to the fact that higher inflation leads to lower mean markups and thus alleviates the distortions from monopolistic competition (compare Figure 2). The monopolistic distortion in goods markets is shut off by Adam and Weber (2019a) with the help of a sales subsidy. A point worth noting is that, in the absence of wage stickiness, the welfare losses, measured in consumption equivalents, are comparably small if an inflation rate of zero rather than the socially optimal inflation rate is chosen.

Table 2 also shows that scenarios that abstract from sticky goods prices or from increases in firm productivity with firm age lead to lower optimal rates of inflation than the main variant of our model. This is to be expected as in these variants of our model the effects in Adam and Weber (2019a) that lead to positive optimal inflation rates are absent. It may be worth stressing that the differences in optimal trend inflation between these variants and our main model are comparably small. Hence the finding in Amano et al. (2009) that wage stickiness is more important than price stickiness for understanding the effects of trend inflation extends to our framework.

A unique feature of our analysis of trend inflation is the age-dependent productivity of workers. Table 2 shows that this feature has non-negligible effects. The particularly high productivity growth for young workers causes optimal inflation to be substantially lower in our main model compared to the scenario where productivity does not depend on the age of a worker.

Next, we examine the consequences of aggregate growth for optimal inflation. This exercise is relevant as the long-term growth rate has declined in many economies and may well remain low in the future (see e.g. Kose and Ohnsorge, 2023). Lower growth makes higher inflation rates optimal. This can be explained by noting that lower aggregate growth also leads to lower increases in worker productivity over time. Lower growth in worker productivity makes the automatic increases in real wages that are brought about by deflation less important.

Finally, our results do not differ substantially from the main model, if we extenuate the parameters governing the harmful labor market effects on young workers. A higher Frisch elasticity implies lower disutility from labor. Lower elasticity of substitution for labor leads to a weaker increase in demand for a specific labor type in case of a wage decline. In both cases considered the consumption equivalents decline compared to our main model, but the socially optimal trend inflation is still well below -2%.

4 Politico-economic Equilibrium

In this section, we analyze transition dynamics and the inflation rate that would be selected by the political process. In particular, we consider the following situation. The economy is in a steady state before period 0. At the beginning of period 0, before workers know whether they are able to adjust their wages in this period, a change in trend inflation is put to a vote, where the central bank's change in policy would be so strong such that the new level of trend inflation would be attained immediately. After this change, there would be no further changes in inflation and the economy would eventually converge to the new steady state. The possibility to change trend inflation is completely unexpected before period 0.

We call a politico-economic equilibrium a situation where a majority of individuals prefer the status quo to both an increase and a decrease in trend inflation in period 0. We consider a grid of possible inflation rates, i.e. -4%, -3%, -2%, ..., 5%, and 6%. Our simulations reveal that the unique politico-economic equilibrium involves an inflation rate of -2%. At this level of deflation, most individuals, except for workers before the beginning of their thirties, are against lowering inflation further, and all individuals oppose higher inflation. This is illustrated in the first column of Figure 8, which shows the utility changes, measured in consumption equivalents, as a function of age for a decrease in inflation to -3% and for an increase to -1%.

Young workers tend to prefer very low rates of inflation for reasons that we have discussed before. Young workers' individual productivities increase more strongly over time compared to the productivities of older workers. Low negative

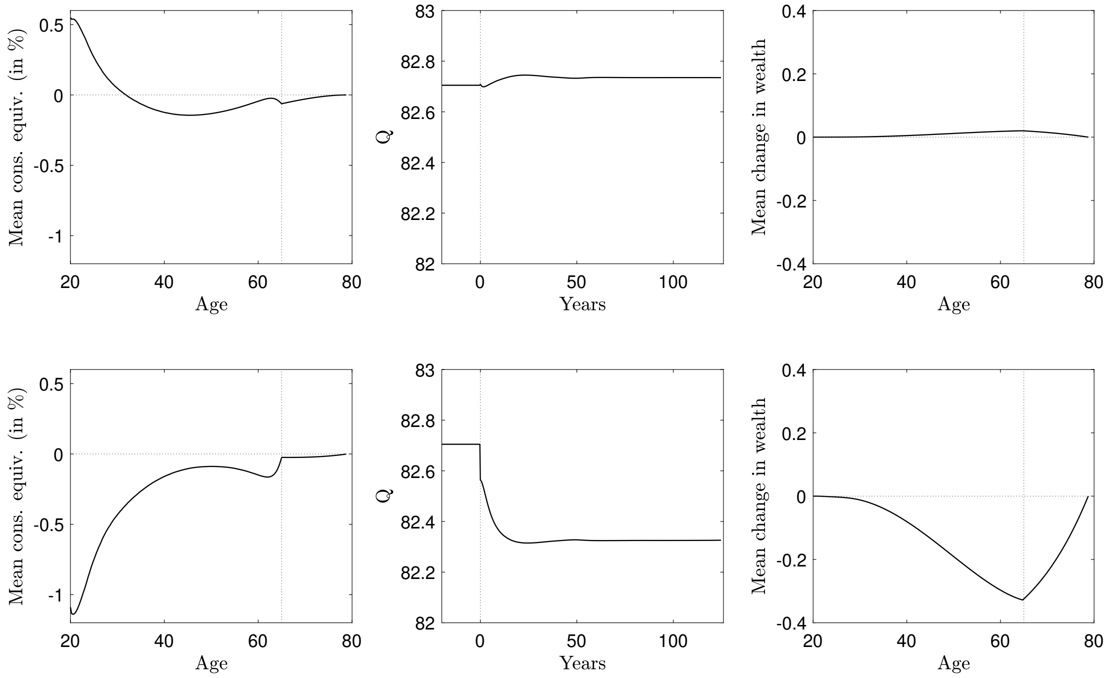

Figure 8: Consequences of a change in trend inflation. First row: transition from -2% to -3%. Second row: transition from -2% to -1%. First column: mean change in individuals' lifetime utility, measured in consumption equivalents, for different age groups if trend inflation was changed to the new level. Second column: transition paths for stock prices in response to an unexpected and permanent change in inflation in period 0. Third column: mean change in individuals' wealth in period 0 for different age groups.

rates of inflation make real wages increase over time automatically even when nominal wages are fixed. This is desirable for young workers as it allows them to choose relatively low wage markups, which leads to high labor income on average. Moreover, the automatic increase in real wages caused by deflation entails smaller changes in nominal wages whenever such changes are possible. Hence individual uncertainty is low for young workers.

One mechanism that has not been discussed so far is the discontinuous change in stock prices brought about by unexpected changes in trend inflation. The time path of stock prices is shown in the second column of Figure 8. While a change from an inflation rate of -2% to -3% only leads to a negligible jump in stock prices, the change from -2% to -1% induces a comparably sizable drop. This drop is compatible with the fact that, compared to an inflation rate of -2%, an inflation rate of -1% entails roughly the same price markups but lower output and thus smaller profits in the long run (see Figure 5). While all individuals invest identical shares of their wealth into stocks as opposed to physical capital, the drop in stock prices affects individuals close to the retirement age of 65 years most strongly because these individuals are the wealthiest.

The utility changes of retirees and workers close to retirement are also affected by changes in real interest rates. This explains why these individuals oppose a drop of inflation to -3%. This drop would lead to moderate gains in wealth but would come at the cost of lower real interest rates in the long run (compare Figure 5). Lower interest rates are harmful to this group of individuals, for whom wealth rather than labor is the most important source of income.

5 Conclusions

This paper has revisited the question of which level of inflation central banks should target. Our model incorporates sticky prices and firm-specific productivity growth. Taken together, these factors have been shown to lead to positive optimal inflation rates of around 2% (Adam and Weber, 2019a; Adam et al., 2022). In addition, our model includes age-specific worker productivity as well as the key ingredients from Amano et al. (2009), namely aggregate productivity growth and sticky wages, which tend to make deflation optimal. Overall we show that the effects that tend to lead to low and even negative optimal inflation rates outweigh those that make positive inflation optimal.

Our analysis has also highlighted conflicts of interest between generations. As the productivity of young workers grows at a high rate, they benefit strongly from deflation. Inflation would require high markups on average as nominally sticky wages pose the risk of real markups that are substantially below their desired levels, which would be very costly to workers. Deflation causes real wages to grow automatically and roughly in line with productivity and is thus desirable.

While older workers and retirees do not benefit from deflation as much as young workers, they still oppose transitions from inflation rates of -2% to higher levels. This is due to the fact that increases in inflation tend to lead to a drop in stock prices, which affects the relatively wealthy individuals the most. As a consequence, the political process leads to inflation rates of -2%, which is higher than the socially optimal one. Our analysis has thus discovered a source of inflation bias that is different from the traditional one that is associated with time-inconsistent policies (Kydland and Prescott, 1977; Barro and Gordon, 1983).

Currently, no central bank intends to target a negative inflation rate of -3.75%. There are two main reasons. First, official measures of inflation are biased upwards. According to recent estimates by Braun and Lein (2021), the bias can be sizable, namely 2.6 percentage points on average and even 3.7 percentage points in the wake of large shocks to relative prices. In the presence of such a measurement bias, the optimal inflation rate suggested by our analysis would correspond to a relatively mild deflation rate of around -1%. Second, central banks are concerned about the effective lower bound for nominal interest rates. Positive inflation targets entail higher nominal interest rates and thus ensure that nominal interest rates are sufficiently far away from the lower bound. As argued before, the ongoing trend towards cashless societies may well render the effective lower bound substantially less relevant in the future. As a consequences, the case for mild deflation may be more compelling in the future.

A Bellman Equations

A.1 Overview

In the following, we specify the optimization problems of intermediate-goods producers and households in a steady state. The optimization problems for the transition dynamics are straightforward extensions. Obviously, the optimization problems during a transition have to take into account that the aggregate variables as well as the distributions of wages and prices change over time.

A.2 Optimization problem of the intermediate-goods producers

Recall that variables with a "$\sim$" are variables that are detrended by dividing them by $\left(a^{\frac{1}{1-\alpha}}\right)^{t}$. Accordingly, we write $\widetilde{Y}_t = Y_t \left(a^{\frac{1}{1-\alpha}}\right)^{-t}$ and $\widetilde{w}_t = w_t \left(a^{\frac{1}{1-\alpha}}\right)^{-t}$, where w_t is the composite real wage W_t/P_t. Moreover, we introduce $p_{f,t} = \frac{P_{f,t}}{P_t}$.

In a steady-state, detrended profits $\widetilde{\Pi}_{i,t}$ can be written as

$$\widetilde{\Pi}_{i,t} = p_{f,t}^{-\varepsilon}\widetilde{Y}\left(p_{f,t} - \frac{1}{X_{f,t}}\frac{r^{\alpha}\widetilde{w}^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\right), \tag{21}$$

where $\frac{1}{X_{f,t}}\frac{r^{\alpha}\widetilde{w}^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}$ is firm f's marginal cost, provided that it uses an optimal capital-labor ratio:

$$\frac{\widetilde{K}_f}{L_f} = \frac{\alpha}{1-\alpha}\frac{\widetilde{w}}{r} \tag{22}$$

With probability $1-\phi$, firms can choose their optimal prices. In this case, the value function $V^F_{adj}(X_{f,t})$ satisfies:

$$\begin{aligned} V^F_{adj}(X_{f,t}) = \max_{p_{f,t}} \Bigg\{ & p_{f,t}^{-\varepsilon}\widetilde{Y}\left(p_{f,t} - \frac{1}{X_{f,t}}\frac{r_t^{\alpha}\widetilde{w}_t^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\right) \\ & + (1-d)\frac{a^{\frac{1}{1-\alpha}}}{1+r_{t+1}-\delta}\mathbb{E}_t V^F(p_{f,t+1}, X_{f,t+1}) \Bigg\} \\ \text{s.t. } X_{f,t+1} &= qX_{f,t}, \\ p_{f,t+1} &= p_{f,t}/\pi, \\ \mathbb{E}_t V^F(p_{f,t+1}, X_{f,t+1}) &= \phi V^F_{nadj}(p_{f,t+1}, X_{f,t+1}) + (1-\phi)V^F_{adj}(X_{f,t+1}), \end{aligned} \tag{23}$$

where "adj" stands for the possibility to adjust ones price. The subscript "nadj" describes situations where firms cannot adjust their prices. If a firm cannot adjusts its nominal price in period $t+1$, its relative price is the previous period's relative price, divided by inflation: $p_{f,t+1} = \frac{p_{f,t}}{\pi}$

For firms that cannot adjust their prices, the value function satisfies

$$\begin{aligned} V^F_{nadj}(p_{f,t}, X_{f,t}) = p_{f,t}^{-\varepsilon}\widetilde{Y}\left(p_{f,t} - \frac{1}{X_{f,t}}\frac{r^\alpha \widetilde{w}^{1-\alpha}}{\alpha^\alpha(1-\alpha)^{1-\alpha}}\right) \\ + (1-d)\frac{a^{\frac{1}{1-\alpha}}}{1+r-\delta}\mathbb{E}_t V^F(p_{f,t+1}, X_{f,t+1}) \\ \text{s.t. } X_{f,t+1} = qX_{f,t}, \\ p_{f,t+1} = p_{f,t}/\pi, \\ \mathbb{E}_t V^F(p_{f,t+1}, X_{f,t+1}) = \phi V^F_{nadj}(p_{f,t+1}, X_{f,t+1}) + (1-\phi)V^F_{adj}(X_{f,t+1}). \end{aligned} \tag{24}$$

A.3 Individuals' decision-making problem

As both assets are perfect substitutes, it is useful to introduce $\widetilde{\Omega}_{i,t}$, which is individual i's detrended level of real wealth at the beginning of period t.

If it is possible to adjust the nominal wage in period t, the worker's problem is

$$\begin{aligned} & V^W_{adj}(\tau_{i,t}, \widetilde{\Omega}_{i,t}) \\ = & \max_{\widetilde{C}_{i,t}, \widetilde{w}_{i,t}, \widetilde{\Omega}_{i,t+1}} \left\{ \ln(\widetilde{C}_{i,t}) - \eta\frac{H_{i,t}^{1+\kappa}}{1+\kappa} + \beta\mathbb{E}_t V^W\left(\tau_{i,t+1}, \widetilde{w}_{i,t+1}, \widetilde{\Omega}_{i,t+1}\right)\right\} \\ & \text{s.t.} \\ & \widetilde{w}_{i,t+1} = \frac{\widetilde{w}_{i,t}}{a^{\frac{1}{1-\alpha}}\pi}, \\ & \tau_{i,t+1} = \tau_{i,t} + 1, \\ & \widetilde{C}_{i,t} + a^{\frac{1}{1-\alpha}}\widetilde{\Omega}_{i,t+1} = (1+r-\delta)\widetilde{\Omega}_{i,t} + \widetilde{w}_{i,t}H_{i,t}, \\ & \widetilde{\Omega}_{i,t+1} \geq 0, \\ & H_{i,t} = \left(G^H_{i,t}\right)^{\theta-1}\left(\frac{\widetilde{w}_{i,t}}{\widetilde{w}}\right)^{-\theta} L_t, \\ & \mathbb{E}_t V^W\left(\tau_{i,t+1}, \widetilde{w}_{i,t+1}, \widetilde{\Omega}_{i,t+1}\right) = \omega V^W_{nadj}\left(\tau_{i,t+1}, \widetilde{w}_{i,t+1}, \widetilde{\Omega}_{i,t+1}\right) \\ & \qquad + (1-\omega)V^W_{adj}\left(\tau_{i,t+1}, \widetilde{\Omega}_{i,t+1}\right). \end{aligned} \tag{25}$$

Workers who cannot adjust their nominal wages face the following problem:

$$\begin{aligned}
&V^W_{nadj}(\tau_{i,t},\widetilde{w}_{i,t},\widetilde{\Omega}_{i,t}) \\
&= \max_{\widetilde{C}_{i,t},\widetilde{\Omega}_{i,t+1}} \left\{ \ln(\widetilde{C}_{i,t}) - \eta \frac{H_{i,t}^{1+\kappa}}{1+\kappa} + \beta \mathbb{E}_t V^W \left(\tau_{i,t+1},\widetilde{w}_{i,t+1},\widetilde{\Omega}_{i,t+1}\right) \right\} \\
&\text{s.t.} \\
&\widetilde{w}_{i,t+1} = \frac{\widetilde{w}_{i,t}}{a^{\frac{1}{1-\alpha}}\pi}, \\
&\tau_{i,t+1} = \tau_{i,t} + 1, \\
&\widetilde{C}_{i,t} + a^{\frac{1}{1-\alpha}}\widetilde{\Omega}_{i,t+1} = (1+r-\delta)\widetilde{\Omega}_{i,t} + \widetilde{w}_{i,t}H_{i,t}, \\
&\Omega_{i,t+1} \geq 0, \\
&H_{i,t} = \left(G^H_{i,t}\right)^{\theta-1}\left(\frac{\widetilde{w}_{i,t}}{\widetilde{w}}\right)^{-\theta} L_t, \\
&\mathbb{E}_t V^W \left(\tau_{i,t+1},\widetilde{w}_{i,t+1},\widetilde{\Omega}_{i,t+1}\right) = \omega V^W_{nadj}\left(\tau_{i,t+1},\widetilde{w}_{i,t+1},\widetilde{\Omega}_{i,t+1}\right) \\
&\qquad\qquad + (1-\omega)V^W_{adj}\left(\tau_{i,t+1},\widetilde{\Omega}_{i,t+1}\right).
\end{aligned} \tag{26}$$

For a worker i who reaches the retirement age TR in period t, the value function in the subsequent period is given by the value functions of retirees, i.e.

$$V^W(TR+1,\widetilde{w}_{i,t+1},\widetilde{\Omega}_{i,t+1}) \;=\; V^R(TR+1,\widetilde{\Omega}_{i,t+1}). \tag{27}$$

The retirees' value function is the outcome of the optimization problem:

$$\begin{aligned}
&V^R(\tau_{i,t},\widetilde{\Omega}_{i,t}) = \max_{\widetilde{C}_{i,t},\widetilde{\Omega}_{i,t+1}} \left\{ \ln \widetilde{C}_{i,t} + \beta V^R(\tau_{i,t+1},\widetilde{\Omega}_{i,t+1}) \right\} \\
&\text{s.t.} \\
&\tau_{i,t+1} = \tau_{i,t} + 1, \\
&\widetilde{C}_{i,t} + a^{\frac{1}{1-\alpha}}\widetilde{\Omega}_{i,t+1} = (1+r-\delta)\widetilde{\Omega}_{i,t}, \\
&\widetilde{\Omega}_{i,t+1} \geq 0.
\end{aligned} \tag{28}$$

The boundary conditions of newborn households is

$$\widetilde{\Omega}_{i,t} \;=\; 0 \qquad \textit{for } \tau_{i,t} = 1.$$

The future value function of individuals in the final periods of their lives is normalized to zero. □

B Algorithm

In this section, we describe how we compute the steady states of our model. The different steps are the following:

1. Fix a value of π.
2. Guess values of r, $\widetilde{w}$, $\widetilde{Y}$.
3. Solve firms' optimization problem via value-function iteration and simulate firm behavior to obtain the distribution of $p_{f,t}$ for all generations of firms.
4. Use the joint distribution of $X_{f,t}$ and $p_{f,t}$ to determine goods-market efficiency A^G via (16).
5. Determine the aggregate profits of firms.
6. Determine the aggregate demand for capital $\widetilde{K}$ and labor L by computing the individual demands for all intermediate-goods firms, for given prices $p_{f,t}$.
7. Use backward induction to determine the policy functions for all age groups of retirees and workers.
8. Simulate the behaviors of workers and retirees to obtain the distribution of individual wages $\widetilde{w}_{i,t}$ and capital supply $\widetilde{K}_{i,t}$.
9. Update guess on r, $\widetilde{w}$, $\widetilde{Y}$:
   - Use individual wages to determine the detrended real wage for composite labor (see equation (10)).
   - Determine detrended aggregate output $\widetilde{Y}$ using equation (14). Aggregate consumption $\widetilde{C}$ is determined by aggregating individual consumption choices.
   - Update r upwards or downwards, depending on whether the demand or the supply of capital are larger.
10. Compare the updated r, $\widetilde{Y}$ and $\widetilde{w}$ with the previous guesses. If the changes are larger than a critical value or if the difference between the demand and supply of capital is larger than a critical value, go back to 3.

□

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